

The Lattice QCD Study of the Three-Nucleon Force

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Abstract. We investigate three-nucleon forces (3NF) from lattice QCD simulations, utilizing the Nambu-Bethe-Salpeter (NBS) wave function to determine two-nucleon forces (2NF) and 3NF on the same footing. Quantum numbers of the three-nucleon (3N) system are chosen to be $(I, J^P) = (1/2, 1/2^+)$ (the triton channel). We consider the simplest geometrical configuration where 3N are aligned linearly with an equal spacing, to reduce the enormous computational cost. Lattice QCD simulations are performed using $N_f = 2$ dynamical clover fermion configurations at the lattice spacing of $a = 0.156$ fm on a $16^3 \times 32$ lattice with a large quark mass corresponding to $m_\pi = 1.13$ GeV. We find repulsive 3NF at short distance.

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INTRODUCTION

One of the hottest topic in nuclear physics and astrophysics these days is the understanding of the properties of 3NF. Actually, there are various phenomena where 3NF may play an important role, e.g., the binding energies of light nuclei [1], the properties of neutron-rich nuclei and the supernova nucleosynthesis [2] and the nuclear equation of state (EoS) at high density relevant to the physics of neutron stars [3, 4].

Despite of its phenomenological importance, microscopic understanding of 3NF is still limited. Pioneered by Fujita and Miyazawa [5], 3NF have been commonly studied from two-pion exchange (2π E) models with the Δ -excitation. However, since 3NF is originated by the fact that a nucleon is not a fundamental particle, it is most desirable to determine 3NF from the fundamental degrees of freedom (DoF), i.e., quarks and gluons, on the basis of quantum chromodynamics (QCD). In this proceeding, we report the calculation of 3NF from first-principle lattice QCD.

As for the 2NF from lattice QCD, an approach based on the NBS wave function has been proposed [6, 7]. Resultant (parity-even) 2NF in this approach are found to have attractive wells at long and medium distances and central repulsive cores at short distance. The method has been extended to the hyperon-nucleon (YN) and hyperon-hyperon (YY) interactions [8, 9, 10, 11, 12]. In this report, we extend the method to the 3N system, and perform the lattice QCD simulations of 3NF in the triton channel, $(I, J^P) = (1/2, 1/2^+)$ [13, 14, 15]. For details of this study, refer to Ref. [15].

FORMALISM

Since the detailed formulation for the 2NF is given in Ref. [7], we discuss the extension to the 3N system. We consider the NBS wave function $\psi_{3N}(\vec{r}, \vec{\rho})$ extracted from the six-point correlator as

$$G_{3N}(\vec{r}, \vec{\rho}, t - t_0) \equiv \frac{1}{L^3} \sum_{\vec{R}} \langle 0 | (N(\vec{x}_1) N(\vec{x}_2) N(\vec{x}_3))(t) \overline{(N' N' N')}(t_0) | 0 \rangle \xrightarrow{t \gg t_0} A_{3N} \psi_{3N}(\vec{r}, \vec{\rho}) e^{-E_{3N}(t-t_0)}, \quad (1)$$

$$\psi_{3N}(\vec{r}, \vec{\rho}) \equiv \langle 0 | N(\vec{x}_1) N(\vec{x}_2) N(\vec{x}_3) | E_{3N} \rangle, \quad A_{3N} \equiv \langle E_{3N} | \overline{(N' N' N')} | 0 \rangle, \quad (2)$$

where E_{3N} and $|E_{3N}\rangle$ denote the energy and the state vector of the 3N ground state, respectively, N (N') the nucleon operator in the sink (source), and $\vec{R} \equiv (\vec{x}_1 + \vec{x}_2 + \vec{x}_3)/3$, $\vec{r} \equiv \vec{x}_1 - \vec{x}_2$, $\vec{\rho} \equiv \vec{x}_3 - (\vec{x}_1 + \vec{x}_2)/2$ the Jacobi coordinates.

With the derivative expansion of the potentials [16], the NBS wave function can be converted to the potentials through the following Schrödinger equation,

$$\left[-\frac{1}{2\mu_r} \nabla_r^2 - \frac{1}{2\mu_p} \nabla_p^2 + \sum_{i < j} V_{2N}(\vec{r}_{ij}) + V_{3NF}(\vec{r}, \vec{\rho}) \right] \psi_{3N}(\vec{r}, \vec{\rho}) = E_{3N} \psi_{3N}(\vec{r}, \vec{\rho}), \quad (3)$$

where $V_{2N}(\vec{r}_{ij})$ with $\vec{r}_{ij} \equiv \vec{x}_i - \vec{x}_j$ denotes the 2NF between (i, j) -pair, $V_{3NF}(\vec{r}, \vec{p})$ the 3NF, $\mu_r = m_N/2$, $\mu_p = 2m_N/3$ the reduced masses. If we calculate $\psi_{3N}(\vec{r}, \vec{p})$ for all \vec{r} and \vec{p} , and if all $V_{2N}(\vec{r}_{ij})$ are obtained by (separate) lattice calculations for genuine 2N systems, we can extract $V_{3NF}(\vec{r}, \vec{p})$ through Eq. (3).

In practice, however, the computational cost is enormous, because of enlarged DoF by the 3N (i.e., 9 quarks) and factorial number of Wick contractions. In order to reduce the cost, we develop several techniques, e.g., taking advantage of symmetries, and employing the non-relativistic limit for the nucleon operator in the source. We further restrict the geometry of the 3N. More specifically, we consider the “linear setup” with $\vec{p} = \vec{0}$, with which 3N are aligned linearly with equal spacings of $r_2 \equiv |\vec{r}|/2$. In this setup, the third nucleon is attached to $(1, 2)$ -nucleon pair with only S-wave. Considering the total 3N quantum numbers of $(I, J^P) = (1/2, 1/2^+)$, the triton channel, the wave function can be completely spanned by only three bases, which can be labeled by the quantum numbers of $(1, 2)$ -pair as 1S_0 , 3S_1 , 3D_1 . Therefore, the Schrödinger equation leads to the 3×3 coupled channel equations with the bases of ψ_{1S_0} , ψ_{3S_1} , ψ_{3D_1} . The reduction of the dimension of bases is expected to improve the S/N as well.

We then consider the identification of genuine 3NF. It is a nontrivial work: Although both of parity-even and parity-odd 2NF are required to subtract 2NF part in Eq. (3), parity-odd 2NF have not been obtained yet in lattice QCD. In order to resolve this issue, we consider the following channel,

$$\psi_S \equiv \frac{1}{\sqrt{6}} \left[-p_\uparrow n_\uparrow n_\downarrow + p_\uparrow n_\downarrow n_\uparrow - n_\uparrow n_\downarrow p_\uparrow + n_\downarrow n_\uparrow p_\uparrow + n_\uparrow p_\uparrow n_\downarrow - n_\downarrow p_\uparrow n_\uparrow \right], \quad (4)$$

which is anti-symmetric in spin/isospin spaces for any 2N-pair. Combined with the Pauli-principle, it is automatically guaranteed that any 2N-pair couples with even parity only. Therefore, we can extract 3NF unambiguously using only parity-even 2NF. Note that no assumption on the choice of 3D-configuration of \vec{r} , \vec{p} is imposed in this argument, and we can take advantage of this feature for future 3NF calculations with various 3D-configuration setup.

LATTICE QCD SETUP AND RESULTS

We employ $N_f = 2$ dynamical configurations with mean field improved clover fermion and RG-improved gauge action generated by CP-PACS Collaboration [17]. We use 598 configurations at $\beta = 1.95$ and the lattice spacing of $a^{-1} = 1.269(14)$ GeV, and the lattice size of $V = L^3 \times T = 16^3 \times 32$ corresponds to $(2.5 \text{ fm})^3$ box in physical spacial size. For u, d quark masses, we take the hopping parameter at the unitary point as $\kappa_{ud} = 0.13750$, which corresponds to $m_\pi = 1.13$ GeV, $m_N = 2.15$ GeV and $m_\Delta = 2.31$ GeV. We use the wall quark source with Coulomb gauge fixing. In order to enhance the statistics, we perform the measurement at 32 source time slices for each configuration, and the forward and backward propagations are averaged. The results from both of total angular momentum $J_z = \pm 1/2$ are averaged as well. We perform the simulation at eleven physical points of the distance r_2 with the linear setup.

In Fig. 1 (left), we plot the radial part of each wave function of $\psi_S = (-\psi_{1S_0} + \psi_{3S_1})/\sqrt{2}$, $\psi_M \equiv (\psi_{1S_0} + \psi_{3S_1})/\sqrt{2}$ and ψ_{3D_1} obtained at $(t - t_0)/a = 8$. Here, we normalize the wave functions by the center value of $\psi_S(r_2 = 0)$. What is noteworthy is that the wave functions are obtained with good precision, which is quite nontrivial for the 3N system. We observe that ψ_S overwhelms the wave function, indicating that higher partial waves are strongly suppressed, and thus the effect of the next leading order in the derivative expansion, spin-orbit forces, is suppressed in this lattice setup.

We determine 3NF by subtracting 2NF from total potentials in the 3N system. Since we have only one channel (Eq. (4)) which is free from parity-odd 2NF, we can determine one type of 3NF. In this report, 3NF are effectively represented in a scalar-isoscalar functional form, which is often employed for the short-range 3NF in phenomenology.

In Fig. 1 (right), we plot the results for the effective scalar-isoscalar 3NF at $(t - t_0)/a = 8$. Here, we include r_2 -independent shift by energies, $\delta_E \simeq 5$ MeV, which is determined by long-range behavior of potentials (2NF and effective 2NF in the 3N system) [15]. While δ_E suffers from $\lesssim 10$ MeV systematic error, it does not affect the following discussions much, since δ_E merely serves as an overall offset. In order to check the dependence on the sink time slice, we calculate 3NF at $(t - t_0)/a = 9$ as well, and confirm that the results are consistent with each other [15].

Fig. 1 (right) shows that 3NF are small at the long distance region of r_2 . This is in accordance with the suppression of $2\pi E$ -3NF by the heavy pion. At the short distance region, on the other hand, an indication of repulsive 3NF is observed. Note that a repulsive short-range 3NF is phenomenologically required to explain the properties of high density matter. Since multi-meson exchanges are strongly suppressed by the large quark mass, the origin of this short-range 3NF may be attributed to the quark and gluon dynamics directly. In fact, we recall that the short-range repulsive (or attractive) cores in the generalized two-baryon potentials are calculated in lattice QCD in the flavor SU(3) limit, and the results are found to be well explained from the viewpoint of the Pauli exclusion principle in the quark level [9]. In this context,

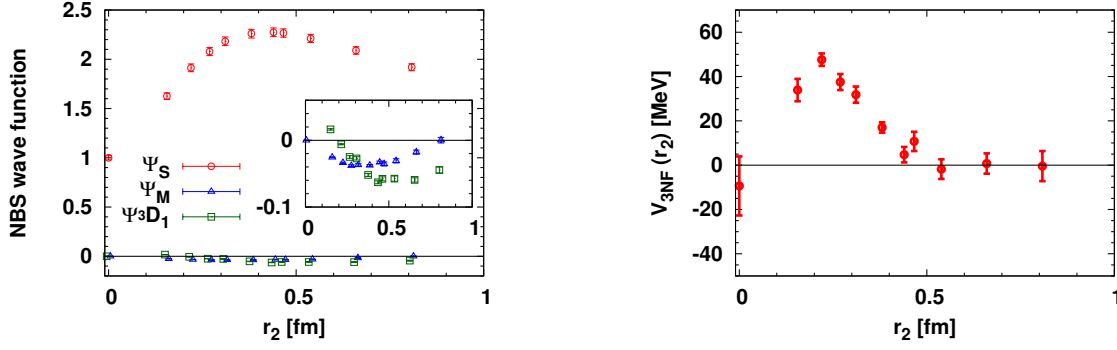


FIGURE 1. (color online). (Left) 3N wave functions at $(t - t_0)/a = 8$. Circle (red), triangle (blue), square (green) points denote Ψ_S , Ψ_M , Ψ^3D_1 , respectively. (Right) The effective scalar-isoscalar 3NF in the triton channel with the linear setup.

it is intuitive to expect that the 3N system is subject to extra Pauli repulsion effect, which could be an origin of the observed short-range repulsive 3NF. Further investigation along this line is certainly an interesting subject in future.

As regards the systematic error, one may worry about the discretization error, since the nontrivial results are obtained at short distance. In particular, the kinetic terms could suffer from a substantial effect, since they are calculated by the finite difference Laplacian operator as $\nabla^2 f(x) = \nabla_{\text{std}}^2 f(x) \equiv \frac{1}{a^2} \sum_i [f(x + a_i) + f(x - a_i) - 2f(x)]$. In order to estimate this artifact, we also analyze using the improved Laplacian operator for both of 2N and 3N, $\nabla_{\text{imp}}^2 f(x) \equiv \frac{1}{12a^2} \sum_i [-(f(x + 2a_i) + f(x - 2a_i)) + 16(f(x + a_i) + f(x - a_i)) - 30f(x)]$. We observe that the results are consistent with each other, and the discretization artifact of 3NF in Laplacian operator is small [15]. Of course, this study probes only a part of discretization errors, and explicit simulations with a finer lattice are on-going.

Since the lattice simulations are carried out only at single large quark mass, quark mass dependence of 3NF is certainly an important issue. In the case of 2NF, short-range cores have the enhanced strength and broaden range by decreasing the mass [7]. We, therefore, would expect a significant quark mass dependence exist in short-range 3NF as well. Quantitative investigation through lattice simulations with lighter quark masses are currently underway.

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